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Book Review

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Generalized Riemann Problems in Computational Fluid Dynamics

Matania Ben-Artzi and Joseph Falcovitz, Cambridge University Press, New York, 2003, 349 pp., $75.00

The generalized Riemann problem, or GRP, is a generalization of Riemann's original initial-value problem (IVP), which poses the question of what happens when two uniform states, meeting at a discontinuity, evolve under the governance of a hyperbolic system of conservation laws such as the Euler equations of fluid dynamics. The generalization simply adds a nonzero first derivative to the spatial distributions of the state quantities on either side of the discontinuity. The spatial derivatives create a nonzero time derivative inside as well as outside of the domain of interaction of the two distributions. For a linear system of equations the exact solution can easily be found by transforming to characteristic state variables, which satisfy uncoupled advection equations. For a single nonlinear conservation law such as the inviscid form of Burgers's equation, the exact solution may also be known explicitly or obtained numerically with arbitrary precision by a root finder. For a nonlinear system the exact solution is not generally known, and different choices of state quantities assumed to vary linearly in space cause substantial solution differences over longer integration times.

The GRP as a subject of research was created by yours truly, although I did not call it GRP, in a 1979 paper,[1] which is Ref. 112 in the book under review. An earlier paper[2] includes a GRP solution for Burgers's equation and an even further generalized Riemann problem, allowing quadratic distributions on both sides of the initial discontinuity. In the 1979 paper, Godunov's first-order method for fluid dynamics, which discretizes a fluid as a sequence of uniform slabs, is extended to second-order accuracy by introducing piecewise-linear subcell distributions of state quantities and then estimating the resulting time derivatives at the cell interfaces. Thus, at each cell interface, a GRP arises, whereas in Goduov's method the original Riemann IVP arises. The 1977 paper does the same for linear advection and for Burgers's equation. The first second-order, Godunov-type code for two-dimensional fluid dynamics, developed in collaboration with Paul R. Woodward, was called MUSCL, which stands for monotone upstream-centered scheme for conservation laws. It contains approximate formulas for the time derivatives at the cell interface in the solution of the local GRP. (The original manuscript of the 1979 paper contained both the approximate formulas and the much more elaborate exact formulas; the latter fell victim to a size reduction ordered by the editors.)

Different MUSCL-type codes developed later by others, including this reviewer, vary in the way the time-centered interface fluxes are computed. My favorite has long been Steve Hancock's 1980 predictor–corrector method, never published by its author but described and tested in a 1982 paper by G. Dick van Albada et al.[3] The authors of the book under review presented, in two Technion reports (1982–1983), a MUSCL scheme incorporating the exact interface time derivatives; they called it the GRP method. The essence of this work appeared in a 1984 journal paper.[4]

Other ways in which MUSCL-type schemes can vary are as follows: the choice of state variables that are assumed to be linearly distributed and the choice of limiter used to preserve solution monotonicity.

So much about the genesis of the subject. The book under review is an elaborate account of the GRP, including a description of the GRP method and numerical applications of the latter. The material includes introductions to hyperbolic equations and their numerical approximation, discussions of wave interactions in compressible flow, and a chapter on reacting flow. Quasi-one-dimensional flow (in a duct of variable cross section) is also covered. The authors do offer an extension to multiple dimensions via dimensional operator splitting; in addition, Chapter 8 on moving grids and discontinuity/boundary tracking is also two dimensional. More two-dimensional material is found in Chapters 3 (Burgers's equation) and 10 (channel flow).

The book can best be described as a monograph with limited scope. The treatment of the GRP focuses on obtaining the initial time derivatives of the state quantities in the interaction zone; nothing is said about the longer-term solution. The treatment of second-order Godunov methods is even more restricted: the authors exclusively concentrate on their own implementation, which uses the interface time derivatives twice, first to center the interface fluxes in time and second to update the interface solution. The time-centered fluxes render second-order accuracy to the scheme while differencing the updated interface values yields a tight approximation to the new solution gradients. This double use of the time derivatives in a second-order, Godunov-type scheme was referred to as "Scheme II" in my 1977 paper,[2] not cited in the book. This scheme has the anomaly, not discussed by the book's authors, that it changes the gradient values by a finite amount even for an infinitesimal time step;

fortunately, gradient limiting usually eliminates the harmful effect of this quirk.

Equally exclusive is the discussion of limiters, indispensable in the practical application of second-order methods for nonlinear equations. Without the availability of limiters, solving the GRP would remain a theoretical exercise. In a book of this scope one would expect a display of the body of knowledge regarding limiting, including total-variation-dimishing schemes and convergence proofs for schemes with limiters. The authors do not seem to want to get involved in these issues. Their steadfast choice of limiter is what I call "double minmod"; they admit (p. 49) this one does not lend itself to convergence proofs, in contrast to minmod, the only other limiter mentioned in the book. Regarding the subject of convergence they refer the reader to the literature cited.

In Abstract C I found evidence that the implementation of the GRP method has not really evolved over the past two decades. The description of the algorithm solving the Riemann problem for an f-law gas closely follows the treatment in van Leer.[1] This includes a geometric interpretation of the Newton iterations used to find the intersection point of Poisson and/or Hugoniot curves in the velocity-pressure plane. This approach has long been abandoned; in practice it is more profitable to write the intersection problem abstractly as $j(p^*) = 0$, where p^* is the pressure resulting in the disturbed region, and then use one's favorite root finder. The visualization in the (u, p)-plane is used only in determining upper and lower bounds for p^*.

In summary, this monograph is a compilation of the authors' previous contributions to the numerical solution of hyperbolic PDEs, centered on the GRP and one particular Godunov-type scheme called the GRP method. It is self-contained in the sense that background material is provided to make the main chapters understandable. The GRP method itself is somewhat dated; there are nonlinear variants around nowadays that implement this approach more efficiently. The authors do not describe these; they have not tried to be comprehensive in the sense of covering related work done by others. The list of references is not comprehensive either. This limits the utility of the book. For example, I would not recommend it as an introduction to the subject of hyperbolic equations and their numerical approximation, nor would I use it as the basis for a course on this subject. But it will provide useful material to the interested reader. Whether the readership will include aerospace engineers looking for practical answers is doubtful; the book will appeal more to applied and numerical mathematicians.

References

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